# Probability of First Upgrade After Third Flight

To find the probability that Sam's first upgrade occurs after the third flight, I need to calculate P(X > 3) where X is the flight number of the first upgrade. This follows a geometric distribution with p = 0.10.

P(X > 3) = P(no upgrade in first 3 flights) = (1-p)³ = (0.90)³ = 0.729

Therefore, the probability that Sam's first upgrade occurs after the third flight is 0.729.

# Probability of Exactly 2 Upgrades in 20 Flights

This follows a binomial distribution with n = 20 and p = 0.10.

P(X = 2) = C(20,2) × (0.10)² × (0.90)¹⁸

P(X = 2) = $\binom{20}{2}$ × (0.10)² × (0.90)¹⁸ = 190 × 0.01 × 0.150 = 0.2852

Therefore, the probability that Sam will be upgraded exactly 2 times in his next 20 flights is approximately 0.285.

# More Than 20 Upgrades in 104 Flights

This can be analyzed using the binomial distribution B(104, 0.10).

The expected number of upgrades = np = 104 × 0.10 = 10.4

Standard deviation = √(np(1-p)) = √(104 × 0.10 × 0.90) = √9.36 = 3.06

The question asks about P(X > 20), which is more than 3 standard deviations above the mean:

(20 - 10.4)/3.06 = 3.14 standard deviations

Using the normal approximation to the binomial (which is appropriate here since np = 10.4 > 5 and n(1-p) = 93.6 > 5), a value that is 3.14 standard deviations above the mean has a probability of approximately 0.00085 (based on the standard normal table or z-score calculation).

Yes, I would be very surprised if Sam received more than 20 upgrades in 104 flights. This outcome is highly unlikely, occurring with probability less than 0.1%, and falls more than 3 standard deviations above the expected value. This would be strong evidence against the airline's claim that the upgrade probability is 0.10.